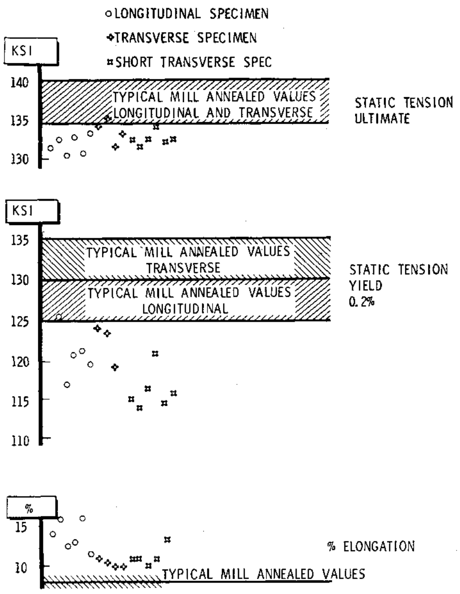

Fig. 3 Roll-diffusion-bonded joint properties, Ti-6Al-4V.

structure from the wrought to cast, locked-in stresses, and the associated distortions which are particularly detrimental in thin gage structure. In brazing and soldering, the joining material has a lower melting temperature and is a weaker material than the parent material; thus, considerable care must be exercised to control the loading pattern. Also, the stress corrosion problem is present, because of the dissimilar materials. In organic bonding, joining material generally has lower strength and limits operating temperature.

The significance of manufacturing imperfections is illustrated by the tests of a 10-ft-diam truss-core sandwich cylinder in bending. The cylinder failed at 80% of the predicted load (62% of the classical buckling load), presumably from the local built-in buckles, as well as over-all out of contour from the theoretical perfect cylindrical shape. The redesign and fabrication of this cylinder to support the desired load level would incur at least a 20% weight increase.

How can the structural designer overcome these difficulties? A new joining process, solid-state diffusion bonding, has received considerable research attention in the past few years. The solid-state diffusion bonding process joins pieces of material together with the characteristics of no joints. The photomicrographs of Fig. 2 show the grain structure across joints achieved by three different diffusion bonding processes. The same parent material grain structure is present across all three joints, thus presenting the desired homogeneous material characteristics across the joints. A cruciform section was designed to establish the engineering properties of the diffusion-bonded joint area. The longitudinal and transverse tensile coupons were used to establish the properties in the basic sheet in the roll-bonded worked area. The short transverse tensile coupons were used to establish the tensile properties across a butt joint.

Figure 3 presents static tension ultimate, yield (0.2%), and percent elongation for roll-diffusion-bonded (60% roll-reduced) 6A1-4V annealed titanium alloy. Of the static tensile specimens, the minimum value of the diffusion bonded area is 97% of the typical mill properties. Of the static tensile yield specimens, the minimum value of the diffusion-bonded area is 91% of the typical mill properties. The percent elongation in the bond area has increased from the typical mill value. These values indicate a joining technique with strength characteristics equal to 97% of the parent material. Compression tests indicate no change in properties from the typical mill properties.

The solid-state diffusion bonding joining process offers the following significant features:

1) Complex structural shapes, such as zees, tees, or "Y"s on a sheet, can be fabricated without a joining penalty.

2) Minimum material usage, through the build-up of bits and pieces of sheet material, instead of carving from a thick plate.

3) Maximum joint efficiency, since the diffusion-bonded joint strength characteristics are approximately equal to the parent material properties and no locked-in stresses are present.

4) Fuel-tight joints, since the joints have the same grain structure as the parent material.

5) Minimization of transverse joints. The current length limitation is the furnace size at the steel mills.

6) Minimization of weight penalty from material sheet tolerances. The original sheet tolerance influence will be reduced by 60% during a 60% roll reduction.

7) True contouring, both single and double curvature, on thin-gage-material complex structural panels, by hot creep forming with the steel matrix material still in place.

8) Minimization of stress risers, since the joint area has the same characteristics as the parent material.

A Statistical Analysis of Yaw Steering for Space Missions

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Nomenclature

a = thrust acceleration
c = effective exhaust velocity
D = matrix composed of d_i variables, $D = \begin{pmatrix} d_1 & d_2 \\ d_3 & -d_1 \end{pmatrix}$
d_1 = $(-c^2/a_0)[(a_0T/c) + \ln(1 - a_0T/c)]$
d_2 = $(-c/a_0)[a_0T^2/2 - d_1]$
d_3 = $c \ln(1 - a_0T/c)$
$E\{\cdot\}$ = expectation operator
$\mathbf{f}$ = thrust acceleration vector, $\mathbf{f} = a\hat{\mathbf{f}}$
I_0 = modified Bessel function of first kind
I_{sp} = specific impulse
K = matrix composed of k_i variables, $K = \begin{pmatrix} k_1 & k_3 \\ k_3 & k_2 \end{pmatrix}$; $k_1 = -d_3/2|D|$, $k_2 = d_2/2|D|$, $k_3 = d_1/2|D|$
L = matrix defined after Eq. (10)
p = function defined in Eq. (12)
r = radius
s = dummy variable
T = burn time or time-to-go
t = time
U = column vector of u_i
u_i = random variables defined in Eq. (9), $i = 1,2$
ΔV = characteristic velocity
W = inverse of covariance matrix of u_1 and u_2, $W = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix}$
Y = column vector of y_0 and $\dot{y}_0$
y = yaw position
z = function defined in Eq. (12)
α, β = constants of integration
θ = coordinate rotation angle defined after Eq. (10)
$\Lambda(p, z)$ = elliptically normal probability function $\left(\int_0^z e^{-s}I_0(sp)ds\right)$

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λ = eigenvalue
μ = gravitational constant
ρ = correlation coefficient
σ = standard deviation

Subscripts

0 = initial condition

Special notes

$(\dot{\ })$ = derivative with respect to time
$(\hat{\ })$ = unit vector

Introduction

FOR many missions, boost vehicle trajectories are designed so that the spacecraft is inserted into a particular plane in space. This is achieved by launching into the desired plane, and using a simple yaw-steering law so that deviations from that plane are nulled at burnout. A yaw-steering law of this type was formulated by Cherry,[1] similar to the explicit solution originally developed by MacPherson.[2] For missions such as satellite interception or lunar impact, however, it is not necessary that the spacecraft be in a particular plane, but only that the plane at burnout contain the target. Teren[3] proposed this type of yaw-steering law based on a calculus-of-variations solution to minimize propellant used for yaw steering. Presumably, this mode of guidance would result in some propellant saving over the method proposed by Cherry. This Note addresses the latter question.

Analysis

If the desired orbital plane is defined by the x-z axes of an inertial coordinate system, then deviations from that plane are in the y direction, and the y equation of motion is

$$\ddot{y} = -(\mu/r^3)y + a\hat{\mathbf{f}}\cdot\hat{\mathbf{y}} \tag{1}$$

The boundary conditions at the burnout time T are $y(T) = 0$, $\dot{y}(T) = 0$. Since time is measured from the present time, T is sometimes called "time-to-go." A closed-form solution to Eq. (1), subject to the prescribed boundary conditions, can be obtained if it is assumed that

$$\hat{\mathbf{f}}\cdot\hat{\mathbf{y}} = \alpha + \beta t + (\mu/ar^3)y \tag{2}$$

where α and β are constants. Equation (2), which is equivalent to the approximation used by MacPherson[2] to obtain his explicit solution, is the yaw-steering law derived by Cherry.[1] Using Eq. (2), Eq. (1) can be integrated to yield

$$\dot{y}(T) = \dot{y}_0 + \int_0^T a(\alpha + \beta t)dt \tag{3a}$$

$$y(T) = y_0 + \dot{y}_0 T + \int_0^T \int_0^t a(\alpha + \beta s)dsdt \tag{3b}$$

where y_0 and $\dot{y}_0$ are the yaw position and yaw velocity, respectively, at the present time.

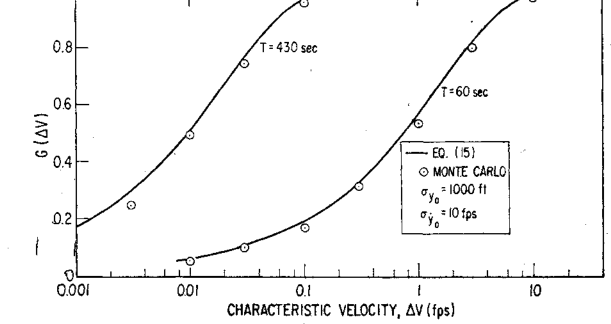

Fig. 1 Probability distribution functions for two different thrusting times.

With a constant-thrust, constant-mass-flow rocket engine, the spacecraft acceleration as a function of time can be written as

$$a = a_0/[1 - (a_0 t/c)] \tag{4}$$

If Eq. (4) is used to evaluate the integrals, Eqs. (3a) and (3b) can be written in matrix form (with the boundary conditions evaluated at T) as

$$Y \equiv \begin{pmatrix} y_0 \\ \dot{y}_0 \end{pmatrix} = D\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{5}$$

Any thrusting in the yaw direction results in a reduction in effective acceleration in the pitch plane, which can be equated to a payload loss.[3] This reduction in effective acceleration is

$$\Delta a = a\{1 - [1 - (\hat{\mathbf{f}}\cdot\hat{\mathbf{y}})^2]^{1/2}\} \approx a[\tfrac{1}{2}(\hat{\mathbf{f}}\cdot\hat{\mathbf{y}})^2]$$

for small accelerations in the yaw direction. The characteristic velocity or loss in effective velocity in the pitch plane is

$$\Delta V \equiv \int_0^T \Delta a dt \approx \int_0^T \frac{1}{2} a(\hat{\mathbf{f}}\cdot\hat{\mathbf{y}})^2 dt \tag{6}$$

To evaluate the integral in Eq. (6), note that the last term in Eq. (2) is the component of the gravitational acceleration in the y direction divided by the thrust acceleration. This term should be very small,[1] so that Eq. (6) can be integrated approximately, using Eq. (4), to yield

$$\Delta V = Y^T K Y \tag{7}$$

where Eq. (5) is used to eliminate the constants α and β. Since ΔV is always positive for all real values of y_0 and $\dot{y}_0$ and is zero only if both y_0 and $\dot{y}_0$ are zero, the aforementioned quadratic form is positive definite, and the K matrix is also positive definite. Therefore, a nonsingular matrix L exists such that

$$L^T K L = I \tag{8}$$

where I is the identity matrix. If new variables u_1 and u_2 are defined by the real transformation,

$$U \equiv \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = L^{-1}Y \tag{9}$$

it is seen that Eq. (7) becomes

$$\Delta V = U^T U = u_1^2 + u_2^2 \tag{10}$$

The L matrix is the product of the orthogonal transformation matrix that diagonalizes K and a diagonal matrix whose elements are functions of the eigenvalues of K. Specifically,

$$L = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \lambda_1^{-1/2} & 0 \\ 0 & \lambda_2^{-1/2} \end{pmatrix}$$

where

$$\theta = \tfrac{1}{2}\tan^{-1}[2k_3/(k_1 - k_2)]$$

$$\lambda_1 = \tfrac{1}{2}\{k_1 + k_2 + [(k_1 - k_2)^2 + 4k_3^2]^{1/2}\}$$

$$\lambda_2 = k_1 + k_2 - \lambda_1$$

Equation (9) shows that the variables u_1 and u_2 are linearly related to y_0 and $\dot{y}_0$. Thus, if y_0 and $\dot{y}_0$ are assumed to be jointly Gaussian random variables with zero means and with variances $\sigma_{y_0}^2$ and $\sigma_{\dot{y}_0}^2$, respectively, then u_1 and u_2 are also jointly Gaussian random variables. The covariance matrix of u_1 and u_2 is obtained from

$$E\{UU^T\} \equiv W^{-1} = L^{-1}E\{YY^T\}(L^T)^{-1}$$

or

$$E\{UU^T\} \equiv W^{-1} = L^{-1}\begin{pmatrix} \sigma_{y_0}^2 & \rho\sigma_{y_0}\sigma_{\dot{y}_0} \\ \rho\sigma_{y_0}\sigma_{\dot{y}_0} & \sigma_{\dot{y}_0}^2 \end{pmatrix}(L^T)^{-1} \tag{11}$$

Both u_1^2 and u_2^2 are central χ-square variables of order one.[4]

Hence, Eq. (10) shows that ΔV is the sum of dependent central χ-square variables. The probability distribution function $G(\Delta V)$ of the sum of dependent χ-square variables is given by[4]

$$G(\Delta V) = \frac{2|W|^{1/2}}{w_{11} + w_{22}} \times \Lambda\left(\frac{[(w_{11} - w_{22})^2 + 4w_{12}^2]^{1/2}}{w_{11} + w_{22}}, \frac{w_{11} + w_{22}}{4}\,\Delta V\right) \quad (12)$$

where $\Lambda(p, z)$ is the elliptically normal probability function tabulated for a range of values of p and z in Refs. 5 and 6. For the missions under investigation in this paper, it can be shown[7] that p is near unity and that z is very large. Using these results, the elliptically normal probability function can be approximated by[7]

$$\Lambda(p,z) \approx \frac{w_{11} + w_{22}}{2|W|^{1/2}} \operatorname{erf}\left\{\left[\frac{|W|\Delta V}{2\,(w_{11} + w_{22})}\right]^{1/2}\right\} \quad (13)$$

Also, the probability distribution function $G(\Delta V)$ can be approximated by[7]

$$G(\Delta V) \approx \operatorname{erf}\left\{\left[\frac{|W|\Delta V}{2(w_{11} + w_{22})}\right]^{1/2}\right\} \quad (14)$$

In terms of more familiar variables, this last equation can be written as

$$G(\Delta V) \approx \operatorname{erf}\left\{\left[\frac{|D|\Delta V}{-d_3\sigma_{y_0}{}^2 + d_2\sigma_{\dot{y}_0}{}^2 + 2d_1\rho\sigma_{y_0}\sigma_{\dot{y}_0}}\right]^{1/2}\right\} \quad (15)$$

Results and Discussion

Some assumptions must be made regarding the booster thrust parameters and the estimate of yaw position and yaw velocity errors. For illustrative purposes, it is sufficient to assume that[3] $a_0 = 26.46$ ft/sec^2, and $c = 13{,}818.7$ fps ($I_{sp} \doteq$ 430 sec). The a priori estimate of the yaw position and yaw velocity errors depends on the specific trajectory being flown. For example, these errors could occur in a satellite intercept mission due to inaccuracies in the orbit determination of the target satellite. For a lunar mission, guidance system hardware errors at parking orbit insertion could result in yaw position and yaw velocity errors at the termination of the parking orbit. Some typical order of magnitude numbers (e.g., see Ref. 8) are $\sigma_{y_0} = 1000$ ft, $\sigma_{\dot{y}_0} = 10$ fps, and $\rho = 0.9$. With these assumptions, $G(\Delta V)$ was computed for two values of burn time T (Fig. 1). The 430-sec burn time is typical of that for a lunar transfer orbit,[3] whereas the 60-sec burn time might be representative of satellite intercept trajectories. Figure 1 shows that the characteristic velocities are generally quite small. For example, with $T = 60$ sec, the probability is 0.99 that ΔV will be less than 12 fps.

The effect of changing σ_{y_0} and $\sigma_{\dot{y}_0}$ is shown in Fig. 2. With $T = 430$ sec, even if the yaw position and velocity standard deviations are 10,000 ft and 100 fps, respectively, the probability is 0.99 that ΔV will be less than 14 fps. However, the assumption of small accelerations in the yaw direction may no longer be valid for the largest standard deviations given in Fig. 2. The effect of other values of correlation coefficient ρ on characteristic velocity can be inferred from Eq. (15). For any constant value of $G(\Delta V)$, increasing ρ requires an increase in ΔV. Since the assumed value for ρ is already near unity, the resulting ΔV cannot be vastly increased by an increase in correlation coefficient.

The effect of other values of c and a_0 can be estimated by expanding the parameters d_1, d_2, and d_3 from Eq. (15) in a power series in a_0T/c. Because $a_0T/c < 1$ [see Eq. (4)], these expansions must converge. For any constant value of $G(\Delta V)$, a decrease in a_0 requires an increase in ΔV. However, it is unlikely that spacecraft accelerations will be significantly smaller than that assumed in this paper. Similarly, an increase in c requires a slight increase in ΔV. But the

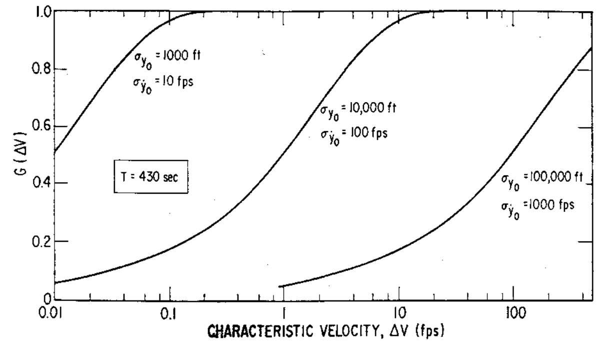

Fig. 2 Effect of yaw error standard deviations on probability distribution function.

assumed c is already near the limit of that attainable by chemical propulsion. Therefore, changing the booster thrust parameters should not materially increase ΔV over the values shown in Figs. 1 and 2.

As a check on the approximations made in this report, a Monte Carlo simulation of the yaw-steering model was made. The random variables y_0 and $\dot{y}_0$ were generated individually for 300 cases, and the probability distribution function was calculated directly. The results, shown in Fig. 1, indicate good agreement between the Monte Carlo simulation and the analytic solution. The small differences are probably due to the approximations in the analysis and to the fact that the distribution of the finite Monte Carlo sample was not quite zero mean Gaussian.

Conclusions

It has been shown that the characteristic velocity required to achieve a particular plane in space is small unless the burn time is short and the yaw position and velocity standard deviations are large. This analysis assumes that the capability exists to launch on time and to launch into the desired plane. Thus, Teren's yaw-steering law,[3] which required more computer storage and a longer calculation time because of its complexity, appears to offer little advantage over the simple yaw-steering law given by Eq. (2). For missions with large yaw deviations and a short burn time, Teren's law would provide some propellant saving (as yet undetermined) together with the practical disadvantages mentioned previously.

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